LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **APRIL 2023**

UST 1502 – PROBABILITY AND DISCRETE DISTRIBUTIONS

Date: 09-05-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION A				
Answer ALL the Questions				
1.	**Define the following.** **(5 x 1 = 5)**			
a)	Classical probability.	K1	CO1
b)	Mutually independent events.	K1	CO1
c)	Conditional probability of bivariate random variable.	K1	CO1
d)	Correlation coefficient.	K1	CO1
e)	Poisson random variable.	K1	CO1
2.	**Answer the following MCQ** **(5 x 1 = 5)**		
a)	The limiting relative frequency approach of probability is known as a. Statistical b. Classical c. Mathematical d. all the above	K1	CO1
b)	If P(A|B)=$\frac{1}{4}$ and P(B|A)=$\frac{1}{3}$, then P(A)/P(B) is equal to ___________ a. 3/4 b. 7/12 c. 4/3 d. 1/12	K1	CO1
c)	The joint distribution function of (X, Y) is equal to the probability_______ a. $P(X = x, Y = y)$ b. $P(X \leq x, Y \leq y)$ c. $P(X \leq x, Y = y)$ d. $P(X \geq x, Y \geq y)$	K1	CO1
d)	If X and Y are two random variables with means $\bar{X}$ and $\bar{Y}$ respectively. Then $E[(X - \bar{X})(Y - \bar{Y}]$ is called: a. $V(X)$ b. $V(Y)$ c. $Cov(X,Y)$ d. Moments of X and Y	K1	CO1
e)	For Bernoulli distribution with probability p of success and q of failure, the relation between mean and variance is: a. mean< variance b. mean> variance c. mean= variance d. mean≤variance	K1	CO1
3.	**Fill in the blanks.** **(5 x 1 = 5)**		
a)	Probability can vary from ______ to ________.	K2	CO1
b)	If A and B are independent events then P(A ∩ B)= _________.	K2	CO1
c)	Two types of random variables are _______ and __________.	K2	CO1
d)	If c is a real value, then $M_{ct}(x) =$ _______.	K2	CO1
e)	Shifting of origin do not affect the _________of the distribution.	K2	CO1
4.	**Match the following.** **(5 x 1 = 5)**			
a)	Pairwise independent	Posterior probability	K2	CO1
b)	Baye's theorem	Bernoulli Distribution	K2	CO1
c)	Continuous random variable	E(X)+E(Y)	K2	CO1

d)	E(X+Y)	Probability density function	K2	CO1
e)	Single trial	P(A ∩ C)=P(A)P(C)	K2	CO1

SECTION B

Answer any TWO of the following questions (2 x 10 = 20)

5.	(i) State and Prove the addition theorem of probability. (ii) A, B and C are three mutually exclusive and exhaustive events associated with a random experiment. Find P(A) such that $P(B) = \frac{3}{2}P(A)$ and $P(C) = \frac{1}{2}P(B)$.	K3	CO2
6.	There are two bags. One bag contains 4 red and 5 black balls and the other 5 red and 4 black balls. One ball is to be drawn from either of the two bags, (i) find the probability of drawing a black ball (ii) Find the probability that the ball is drawn from bag 1 and bag 2 respectively.	K3	CO2
7.	Define the Moment Generating Function and its usage. Also, discuss the properties of MGF.	K3	CO2
8.	If a discrete random variable has the probability function as	K3	CO2

(X=x)	0	1	2	3	4	5	6	7	8
P(X=x)	k	2k	3k	5k	5k	4k	3k	2k	k

i) Calculate the value of k.

ii) Find E(X) and V(X).

SECTION C

Answer any TWO of the following in 100 words (2 x 10 = 20)

9.	Prove that $E(X_1 + X_2 + \cdots + X_n) = E(X_1) + E(X_2) + \cdots E(X_n)$.	K4	CO3
10.	From a city population, the probability of selecting (i) a male or a smoker is 7/10 (ii) a male smoker is 2/5 iii) a male, if already he is a smoker is 2/3. Find the probability of selecting a) a non-smoker b) a male c) a smoker, if a male is first selected.	K4	CO3
11.	Define covariance and discuss its properties.	K4	CO3
12.	Explain the correlation coefficient and its properties.	K4	CO3

SECTION D

Answer any ONE of the following in 250 words (1 x 20 = 20)

13.	i) State and Prove Bayes theorem. ii) Consider three urns containing white(W), black(B) and red (R) balls as follows: Urn I – 2W, 3B and 4R balls; Urn II – 3W, 1B and 2R balls; Urn III – 4W, 2B and 5R balls. Two balls are drawn from an urn and they happen to be one white and one red ball. Find the probability that the two balls 1W and 1R are drawn from urn I, urn II and urn III respectively.	K5	CO4
14.	With usual notations, find p for a binomial variate X, if n=6 and $9\,P(X = 4) = P(X = 2)$.	K5	CO4

SECTION E

Answer any ONE of the following in 250 words (1 x 20 = 20)

15.	For the joint probability distribution of two random variables X and Y given	K6	CO5

	below:		

X	Y			
	1	**2**	**3**	**4**
1	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$
2	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{3}{36}$	$\frac{2}{36}$
3	$\frac{5}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
4	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{5}{36}$

i) Find the marginal distributions of X and Y.

Calculate the conditional distribution of X given Y=1 and the Conditional distribution of Y given X=2.

16.	Define a Binomial random variable. Also, derive the mean and variance of Binomial distribution.	K6	CO5

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